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Upon Electric Charge Systems Moving Without Acceleration and Associated with Charge Creation

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Abstract

A discussion is given of certain electromagnetic fields associated with charges moving with zero acceleration. These fields involve zero magnetic field and the creation of charge.

1. *Introduction*

Recently, Chambers (1970) discussed the motion of charge systems moving with the velocity of light which had associated with them zero magnetic fields and charge creation. It is the purpose of this paper to show that similar solutions exist, when the velocity of the charge distribution has associated with it zero acceleration. An example of such a velocity distribution is Milne's Isotropic Universe (Milne, 1935).

If the magnetic field is zero, the governing equations follow from Chambers (1970). Maxwell's equations become

$$-\frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} - \nabla N \tag{1.1a}$$

$$\nabla . \mathbf{D} = \rho + \frac{1}{c^2}\frac{\partial N}{\partial t} \tag{1.1b}$$

$$\nabla \times \mathbf{E} = 0 \tag{1.1c}$$

The energy density of the field is

$$\tfrac{1}{2}(\epsilon_0 E^2 + \mu_0 N^2) \tag{1.2a}$$

The Poynting vector is

$$-\mathbf{E}N \tag{1.2b}$$

and the equation of motion of a charged particle is

$$\frac{d\mathbf{p}}{dt} = q(\mathbf{E} + \mu_0 \mathbf{v} N) \tag{1.2c}$$

The rate of charge creation per unit volume is given by

$$\nabla^2 N - \frac{1}{c^2}\frac{\partial^2 N}{\partial t^2} \tag{1.2d}$$

Previously (Chambers, 1970) the problem was considered of a charge system which was moving with the velocity of light. In this way the rate of charge of momentum became zero and it became possible to write

$$\mathbf{0} = \mathbf{E} + \mu_0 \mathbf{v} N \tag{1.3a}$$

When this happens, it follows from equation (1.1b) that

$$\rho = -\frac{1}{c^2}\left\{\nabla.(N\mathbf{v}) + \frac{\partial N}{\partial t}\right\} \tag{1.3b}$$

and substituting in equation (1.1a) a differential equation for N follows.

$$\frac{1}{c^2}\left[\frac{\partial}{\partial t}(N\mathbf{v}) + \nabla.(N\mathbf{v})\mathbf{v} + \frac{\partial N}{\partial t}\mathbf{v}\right] + \nabla N = 0 \tag{1.3c}$$

The Poynting vector becomes

$$\mathbf{S} = \mu_0 \mathbf{v} N^2 \tag{1.3d}$$

and the energy density

$$W = \tfrac{1}{2}\mu_0 N^2\left(1 + \frac{v^2}{c^2}\right) \tag{1.3e}$$

The total force on the charge current system within a volume is given by

$$\int d\mathscr{S}.\mathbf{T}^e + \int d\mathscr{S}\frac{\mu_0 N^2}{2} - \frac{1}{c^2}\frac{\partial}{\partial t}\int \mathbf{E}N\,d\tau \tag{1.3f}$$

However, charge distributions moving with the velocity of light are not the only ones for which the acceleration is zero. Writing

$$\mathbf{p} = m\mathbf{v} \tag{1.4a}$$

The rate of charge of momentum is given by

$$\frac{d(m\mathbf{v})}{dt} = m\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v}\right) \tag{1.4b}$$

In this case m is the mass of the particle and the velocity $\mathbf{v}$ is measured in a frame moving with the particle. In this way m does not change, and the full derivative of the velocity must be used (Chambers, 1969).

It follows, therefore, that if $\mathbf{v}$ is such that

$$\frac{d\mathbf{v}}{dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} = 0 \tag{1.4c}$$

the relation (1.3a) holds together with its consequences.

2. *Kinematically Isotropic Flow*

Consider an kinematically isotropic flow of the type introduced by Milne. In this

$$\mathbf{v} = \mathbf{r}/t \tag{2.1}$$

where $\mathbf{r}$ is the position vector as measured from some origin. This satisfies the condition

$$\frac{d\mathbf{v}}{dt} = 0$$

The differential equation for N becomes, if it is assumed that N depends only on r and t

$$\frac{1}{c^2}\left[\frac{\partial}{\partial t}\left(\frac{Nr}{t}\right) + \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^3 N}{t}\right)\cdot\frac{r}{t} + \frac{\partial N}{\partial t}\frac{r}{t}\right] + \frac{\partial N}{\partial r} = 0 \tag{2.2}$$

This becomes

$$\frac{1}{c^2}\left[\frac{\partial N}{\partial t}\cdot\frac{r}{t} - \frac{Nr}{t^2} + \frac{3Nr}{t^2} + \frac{\partial N}{\partial r}\frac{r^2}{t^2} + \frac{\partial N}{\partial t}\frac{r}{t}\right] + \frac{\partial N}{\partial r} = 0$$

or on further simplification

$$\frac{1}{c^2}\left[2\frac{\partial N}{\partial t}\frac{r}{t} + 2\frac{Nr}{t^2} + \frac{\partial N}{\partial r}\frac{r^2}{c^2}\right] + \frac{\partial N}{\partial r} = 0 \tag{2.3}$$

The charge density is

$$-\frac{1}{c^2}\left[\frac{3N}{t} + \frac{\partial N}{\partial r}\frac{r}{t} + \frac{\partial N}{\partial t}\right] \tag{2.4a}$$

and the current density vector is

$$\mathbf{f}\left(\frac{\partial N}{\partial r}\right) + \frac{1}{c^2}\frac{\partial}{\partial t}(N\mathbf{v}) = \mathbf{f}\frac{\partial N}{\partial r} + \frac{1}{c^2}\frac{\partial N}{\partial t}\frac{\mathbf{r}}{t} - \frac{N\mathbf{r}}{c^2 t^2} \tag{2.4b}$$

The charge creation rate per unit volume is given by

$$q = \nabla^2 N - \frac{1}{c^2}\frac{\partial^2 N}{\partial t^2} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial N}{\partial r}\right) - \frac{1}{c^2}\frac{\partial^2 N}{\partial t^2} \tag{2.4c}$$

Two simple functions can be verified directly to be solutions. The first is

$$K/t \tag{2.5a}$$

and the second is

$$N = N_0\{1 - r^2/(c^2 t^2)\} \tag{2.5b}$$

For the first

$$\rho = \frac{-2K}{c^2 t^2} \tag{2.6a}$$

$$\mathbf{J}=\frac{-2K\mathbf{r}}{c^2t^2} \tag{2.6b}$$

$$q=\frac{-4K}{c^2t^3} \tag{2.6c}$$

$$\mathbf{S}=\mu_0\frac{K^2\mathbf{r}}{t^3} \tag{2.6d}$$

$$W=\tfrac{1}{2}\mu_0\frac{K^2}{t^2}\left(1+\frac{r^2}{c^2t^2}\right) \tag{2.6e}$$

This solution is not of any particular interest.

The second solution produces results, however, which may possibly have physical meaning

$$\rho=\frac{-3}{c^2t}N_0\left(1-\frac{r^2}{c^2t^2}\right)=\frac{-3N}{c^2t} \tag{2.7a}$$

$$\mathbf{J}=\frac{-3}{c^2t^2}N_0\left(1-\frac{r^2}{c^2t^2}\right)=\frac{-3N\mathbf{r}}{c^2t^2} \tag{2.7b}$$

$$q=\frac{6N_0}{c^2t^2}\left\{1-\frac{r^2}{c^2t^2}\right\}=-\frac{6N}{c^2t^2}=\frac{2\rho}{t} \tag{2.7c}$$

$$\mathbf{S}=\mu_0N_0^2\left\{1-\frac{r^2}{c^2t^2}\right\}^2\frac{\mathbf{r}}{t}=\mu_0\frac{N^2\mathbf{r}}{t} \tag{2.7d}$$

$$W=\tfrac{1}{2}\mu_0N_0^2\left\{1-\frac{r^2}{c^2t^2}\right\}^2\left\{1+\frac{r^2}{c^2t^2}\right\}=\tfrac{1}{2}\mu_0N^2\left(1+\frac{r^2}{c^2t^2}\right) \tag{2.7e}$$

The total force on the charge current system within a sphere of radius r is given by

$$\int d\mathscr{A}.\mathbf{T}^e+\int d\mathscr{A}\frac{\mu_0N^2}{2} \tag{2.7f}$$

the third term in the expression (1.3f) vanishing by symmetry.

This last equation may be, as indicated previously (Chambers, 1970), interpreted as follows:

(i) a tension per unit area $\frac{1}{2}\epsilon_0E^2=\frac{1}{2}\epsilon_0\mu_0^2N^2v^2$ in the direction radially out,

(ii) a pressure $\frac{1}{2}\epsilon_0\mu_0^2N^2v^2$ in the transverse directions,

(iii) an isotropic pressure $\frac{1}{2}(\mu_0N^2)$.

It will be seen that all these quantities will vanish when N vanishes.

It can also be seen that a possible solution is going to be given by N having the form $N_0(1-r^2/c^2t^2)$ for $r<ct$, and N vanishing for $r>ct$, as for this distribution all the quantities of interest mentioned are zero, and continuous across the sphere $r=ct$.

In particular there is no energy or current flow across $r = ct$, and on $r = ct$ the stresses are zero. Furthermore, for this system

$$v = r/t < c \tag{2.8}$$

always whenever there are non-zero fields, etc. Outside the sphere $r = ct$, the value of v is not meaningful as there is nothing there.

A possible interpretation of this may be given in cosmological terms as follows:

There is an electrically charged universe which obeys the kinematically isotropic condition. This universe is continually expanding and its boundary is given by a sphere of radius ct. The expansion of the universe is caused by a continuous charge creation.

References

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